

On the possibility of q -scaling in high energy production processes

Maciej Rybczyński, Zbigniew Włodarczyk

Institute of Physics, Jan Kochanowski University, Świętokrzyska 15, 25-406 Kielce, Poland

E-mail: Maciej.Rybczynski@ujk.edu.pl, zbigniew.wlodarczyk@ujk.kielce.pl

Grzegorz Wilk

National Centre for Nuclear Research, Hoża 69, 00-681 Warsaw, Poland

E-mail: wilk@fuw.edu.pl

Abstract. It has recently been noticed that transverse momenta (p_T) distributions observed in high energy production processes exhibit remarkably universal scaling behaviour. This is the case when a suitable variable replaces the usual p_T . On the other hand, it is also widely known that transverse momentum distributions in general follow a power-like Tsallis distribution, rather than an exponential Boltzmann-Gibbs one, with a (generally energy dependent) nonextensivity parameter q . Here we show that it is possible to choose a suitable variable such that *all* the data can be fitted by the *same* Tsallis distribution (with the same, energy independent value of the q -parameter). They thus exhibit q -scaling.

PACS numbers: 05.90.+m, 13.85.-t, 11.80.Fv, 13.75.Cs

Almost fifty years ago Hagedorn developed a statistical description of momentum spectra observed experimentally [1]. It predicts an exponential decay of differential cross sections

$$E \frac{d^3\sigma}{d^3p} \simeq C \cdot \exp\left(-\frac{p_T}{T}\right) \quad (1)$$

for transverse momenta, whereas in experiments one observes non-exponential behaviour for large transverse momenta. Hagedorn then proposed the 'QCD inspired' empirical formula describing the data of the invariant cross section of hadrons as a function of p_T over a wide range [2]:

$$E \frac{d^3\sigma}{d^3p} = C \cdot \left(1 + \frac{p_T}{p_0}\right)^{-\alpha} \longrightarrow \begin{cases} \exp\left(-\frac{\alpha p_T}{p_0}\right) & \text{for } p_T \rightarrow 0, \\ \left(\frac{p_0}{p_T}\right)^\alpha & \text{for } p_T \rightarrow \infty, \end{cases} \quad (2)$$

with C , p_0 and α being fit parameters. This becomes pure exponential for small p_T and pure power law for large p_T ‡.

When looking at p_T spectra (1) of secondaries produced in high energy multiparticle processes, it is commonly assumed that the temperature T of the hadronizing system, when treated as a statistical ensemble, can be connected with the observed mean transverse momentum $\langle p_T \rangle$. Usually, however, the system is far from thermal equilibrium and the individual event temperature T cannot correspond to the mean transverse momenta. The temperature fluctuates from event to event (or also in the same event). Such a situation is described by a nonextensive generalization of statistical mechanics proposed quite some time ago [5]. There is one new parameter, q , in addition to the temperature T , and the main formula of interest here is the Tsallis distribution,

$$h_q(p_T) = C_q \cdot \left[1 - (1 - q)\frac{p_T}{T}\right]^{\frac{1}{1-q}} \xrightarrow{q \rightarrow 1} h(p_T) = C_1 \cdot \exp\left(-\frac{p_T}{T}\right), \quad (3)$$

(where C_q is a normalization constant). This coincides with Eq. (2) for

$$\alpha = \frac{1}{q-1} \quad \text{and} \quad p_0 = \frac{T}{q-1}. \quad (4)$$

This approach has been shown to be very successful in describing very different physical systems [5]. Among them are also multiparticle production processes of a different kind (see [6, 7] for recent reviews). The basic conceptual difference between (2) and (3) is in the underlying physical picture. In (2) the small p_T region is governed by *soft physics* possibly described by some unknown nonperturbative theory or model, and the large p_T region is governed by *hard physics* believed to be described by perturbative QCD. In (3), the nonextensive formula is valid in the whole range of p_T and it does not claim to originate from any particular theory. It is just a generalization of the usual statistical mechanics and merely offers the kind of general unifying principle, namely the existence of some kind of complicated equilibrium (or steady state) involving all scales of p_T , which is described by two parameters, T and q . The temperature T describes its mean properties and the parameter q , known as the *nonextensivity parameter*, describes action

‡ Actually this QCD inspired formula was proposed earlier in [3, 4].

of the possible nontrivial long range effects believed to be caused by fluctuations (but also by some correlations or long memory effects) [5]. In fact, it was shown in [8] that q is directly connected to the variance of T ,

$$q = 1 + \frac{\text{Var}(T)}{\langle T \rangle^2}, \quad (5)$$

and therefore describes its intrinsic fluctuations. For $q = 1$ one recovers the usual BG distribution (Eq. (1)). In other words, the widely used thermal bath concept fails to satisfy conditions allowing us to introduce the notion of thermal equilibrium in the BG sense: it is always finite and can hardly be considered homogenous. In fact, in many cases it only occupies a fraction of the allowed phase space or even has a fractal-like structure. In such cases, a heat bath cannot be described by a single parameter T . One has to extend the parameter space to account for all effects mentioned above and use Eq. (3) with a new additional parameter q .

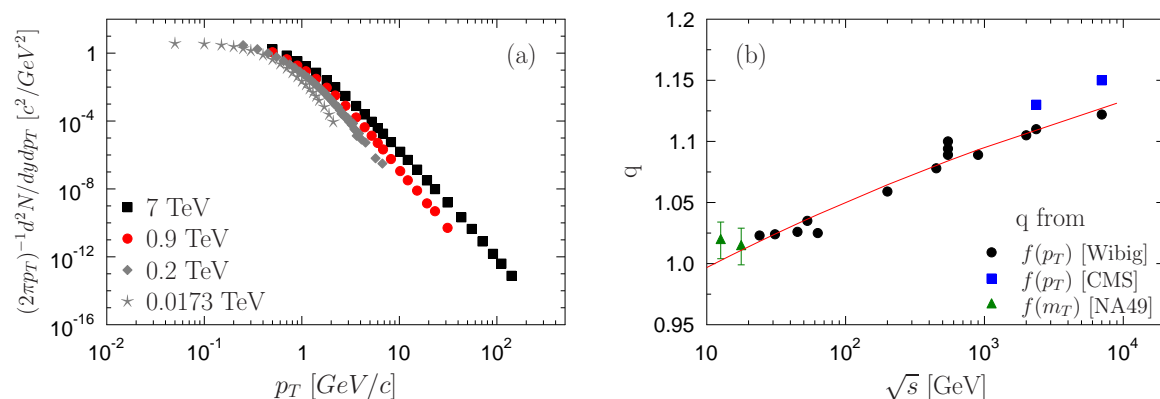


Figure 1. (Color online)(a) Transverse momenta distributions considered by us. (b) The corresponding values of the parameter q obtained from Tsallis fits. Data are from compilation by Wibig [19], NA49 [17], UA1 [18] and CMS [13].

In what follows we concentrate on data on p_T distributions of secondaries produced in $p + \bar{p}$ and $p + p$ collisions at selected energies (covering the wide range of available energies at roughly the same distance in logarithmic scale): 7 TeV and 0.9 TeV from CMS [13], 200 GeV from UA1 [18] and 17.3 GeV from NA49 [17], cf. Fig. 1a. They can be fitted at each energy by using Eq. (3) with $h(p_T) = dN/dp_T^2$. From these fits one finds values of q for different energies, $q(s)$ shown in Fig. 1b. It can be represented by $q(s) = 4/3 - 0.4(\sqrt{s})^{-0.075}$ (full line). The $q(s)$ from Fig. 1b can be translated to

§ Cf. [6, 7] for further references concerning specific applications of this approach to hadronic and nuclear physics in last decade. Recent examples of power-like distributions fitted by nonextensive Tsallis formula are provided in [9, 10], by PHENIX [11] and STAR [12] experiments at RHIC and CMS [13], ATLAS [14] and ALICE [15] experiments at LHC. Finally, the possible QCD origin of such fluctuations and/or correlations could probably be traced back to the nonperturbative QCD, for example, [16].

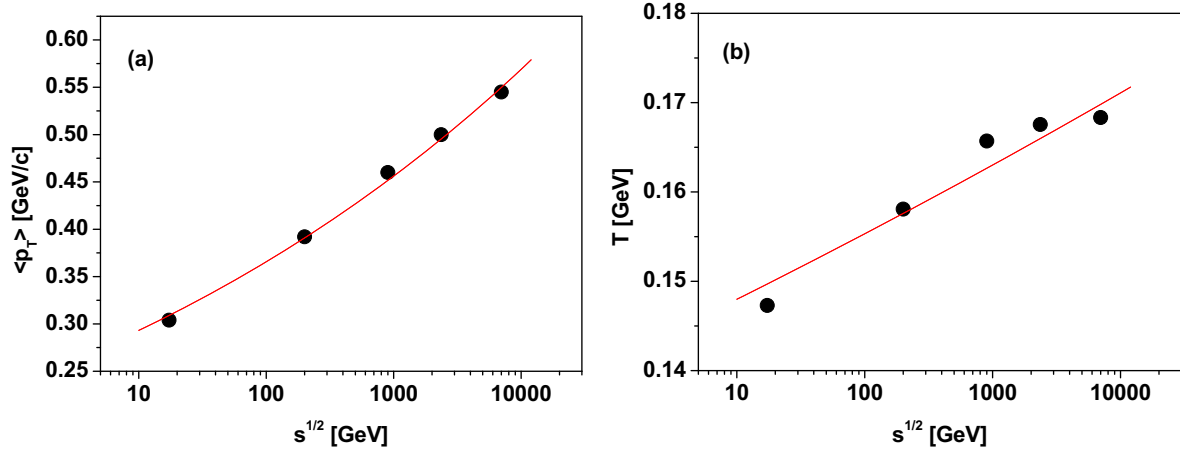


Figure 2. (Color online) (a) Experimental values of mean transverse momenta, $\langle p_T(s) \rangle$, of charged particles produced in $p + p$ and $p + \bar{p}$ collisions [13, 17, 18] (cf. Fig. 1a for corresponding distributions at selected energies). (b) The resulting $T(s)$ as given by Eq. (6).

energy dependence of the temperature, $T(s)$, with the help of $\langle p_T(s) \rangle$,

$$\langle p_T \rangle = \frac{T}{4 - 3q}, \quad (6)$$

Using $\langle p_T(s) \rangle$ as evaluated experimentally, cf. Fig. 2a (it can be parameterized by $\langle p_T(s) \rangle = 0.235 (\sqrt{s})^{0.096}$) one gets the $T(s)$ presented in Fig. 2b, which can be represented by $T(s) = 0.141 (\sqrt{s})^{0.021}$. To summarize: all p_T data considered here can be fitted with a Tsallis formula, Eq. (3), by using an energy dependent parameter $q(s)$ and $T(s)$.

Recently there have been attempts to use an energy independent q to describe dN/dp_T and to check for possible scaling behaviour in p_T [20]. It was found that to this end one has to use a p_T -dependent form of the nonextensivity parameter,

$$q(p_T) = \frac{q_0 - (q_0 - 1) \theta(p_T)}{1 - (q_0 - 1) \theta(p_T)}, \quad (7)$$

where $q_0 = 1.12 \pm 0.06$ and $\theta(p_T) = 2 \log [\log (1 + \kappa p_T)]$ with $\kappa = 0.013 \text{ GeV}^{-1}$. It was shown that any experimentally accessible q depends on p_T growing from $q = 1$ for $p_T = 0$ and attaining $q = q_0$ for $p_T = 9/\kappa$.

The distributions dN/dp_T shown in Fig. 1a differ for different energies. However, as shown in [21, 22, 23], one can find a single scaling function $F(\tau)$, independent of energy, and a suitable scaling variable τ such that one observes a scaling: $h(p_T, \sqrt{s}) \rightarrow F(\tau = f(p_T, \sqrt{s}))$, analogous to Feynman or KNO scaling [24]. Prompted by *geometrical scaling* behaviour found in deep inelastic scattering data, the following universal variable has been proposed [23],

$$\tau = \frac{p_T^2}{Q_{sat}^2}, \quad \text{with} \quad Q_{sat}^2(p_T) = Q_0^2 \left(\frac{p_T}{W} \right)^{-\lambda}. \quad (8)$$

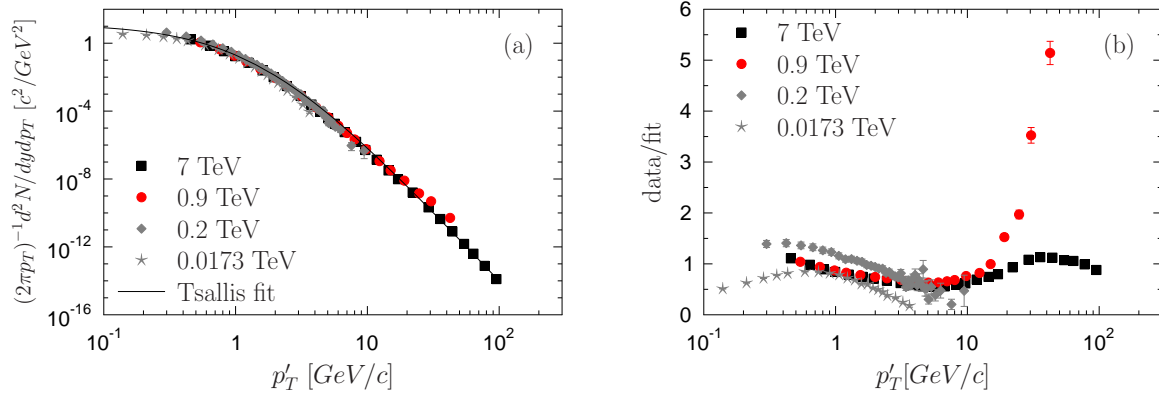


Figure 3. (Color online) (a) Data for transverse momentum distributions for different energies [17, 18, 13] plotted by using the scaling variable p'_T defined in [23] and fitted using Eq. (3). (b) The ratio data/fit for the results presented in (a).

In this variable all data lie on a single curve,

$$\frac{dN}{dp_T^2} = \frac{1}{Q_0^2} F(\tau), \quad (9)$$

with $F(\tau)$ being some energy independent universal function (cf. [23] for details) and λ is a parameter. Actually, to get good agreement with all available data, λ has to depend on p_T . The best fit, see Fig. 3, is obtained with $\lambda = \lambda_{eff}(Q) = 0.13 + 0.1 (Q^2/10)^{0.35}$, where $Q = 2p_T$ (cf., Eq. (11) of [23]). The variable p'_T used in Fig. 3 was obtained by demanding that p_T at energy W should be connected with p'_T at energy W' via the following relation (cf. [23] for details),

$$p'_T = p_T \left(\frac{W'}{W} \right)^{\frac{\lambda}{\lambda+2}}. \quad (10)$$

Fig. 3 shows the corresponding results together with a Tsallis fit performed using Eq. (3) with $C = 14.0$, $q = 1.121$, $T = 0.18$ GeV.

Here we would like to check whether already considered data show a kind of q -scaling as seen from the perspective of Tsallis statistics (and, if so, in what variable). In other words: is it possible to find, in the framework of the nonextensive statistics, a variable (different from the p'_T above) which would scale the dN/dp_T distributions? And, is the parameter $q = 1.121$ from the Tsallis fit in Fig. 3a already universal, or else can one also have such scaling behaviour for some other value of the parameter q using a different scaling variable?

Prompted by KNO scaling as observed in multiplicity distributions [24] we first plot data from Fig. 1a using the scaled transverse momentum variable,

$$z = \frac{p_T}{\langle p_T \rangle}. \quad (11)$$

As seen in Fig. 4, already this variable seems to be nearly satisfactory, except for the largest LHC energies. Agreement with data can be further improved by using Tsallis

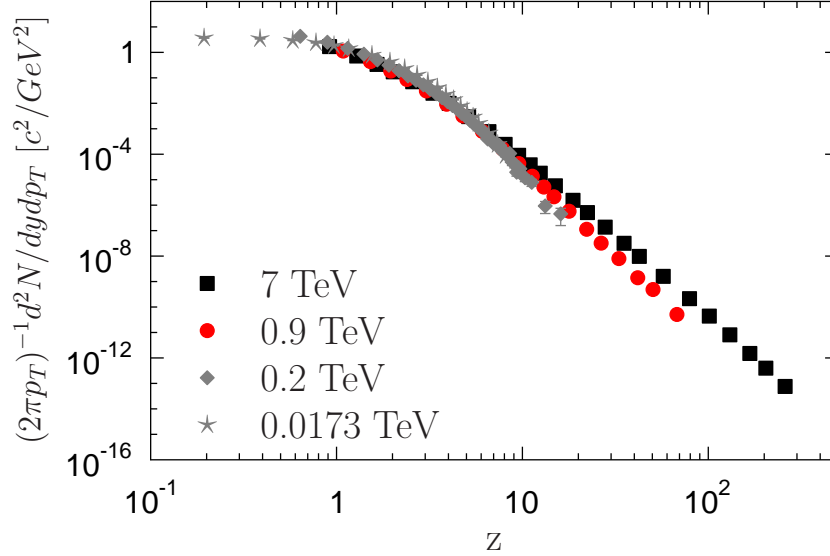


Figure 4. (Color online) Data for transverse momentum distributions for different energies [17, 18, 13] plotted by using a scaling variable z as defined in Eq. (11).

distribution, $h_q(u)$, as given by Eq. (3), in which p_T/T is replaced by u/u_0 , where^{||}

$$u = \frac{p_T}{\langle p_T \rangle - b \cdot p_T}. \quad (12)$$

Using $\langle p_T(s) \rangle$ taken from an experiment as shown in Fig. 2a, and an energy dependent coefficient $b(s) = -0.0397 + 0.08 (\sqrt{s})^{-0.075}$, one can reasonably well fit the data, cf., Fig. 5, with a constant, energy independent value of $q = 1.172$ (and with $C = 79.4$ and $u_0 = 0.17$). Notice that our result is essentially of the same quality as that obtained from the geometrical scaling prescription proposed in [23] (cf., Fig. 3). Both in Fig. 3 and in Fig. 5 there are deviations from Tsallis distributions. Essential here are the differences between different energies. To show this in both cases we evaluate the ratios

$$R = \frac{f(p_T, \sqrt{s})}{f(p_T, \sqrt{s} = 7 \text{ TeV})} \quad (13)$$

of experimental distributions, $f(p_T, \sqrt{s}) = \frac{d^3 N}{2\pi p_T dp_T dy}$ for different energies \sqrt{s} , which are shown in Fig. 6.

To justify using the variable u as defined in Eq. (12), note that one can write $u/u_0 = p_T/T_{eff}$ where T_{eff} is an effective temperature,

$$T_{eff} = T_0 + T_v(p_T); \quad \text{with} \quad T_0 = u_0 \cdot \langle p_T \rangle; \quad T_v = -b \cdot u_0 \cdot p_T. \quad (14)$$

This temperature could be related to the possible p_T transfer, additional to that resulting from a hard collision, perhaps proceeding by a kind of multiple scattering process,

^{||} Notice that u is just a power series in the scaling variable z defined in Eq. (11), $u = \frac{z}{1-b \cdot z} = \sum_{k=1}^{\infty} b^{k-1} z^k = z + bz^2 + b^2 z^3 + \dots$.

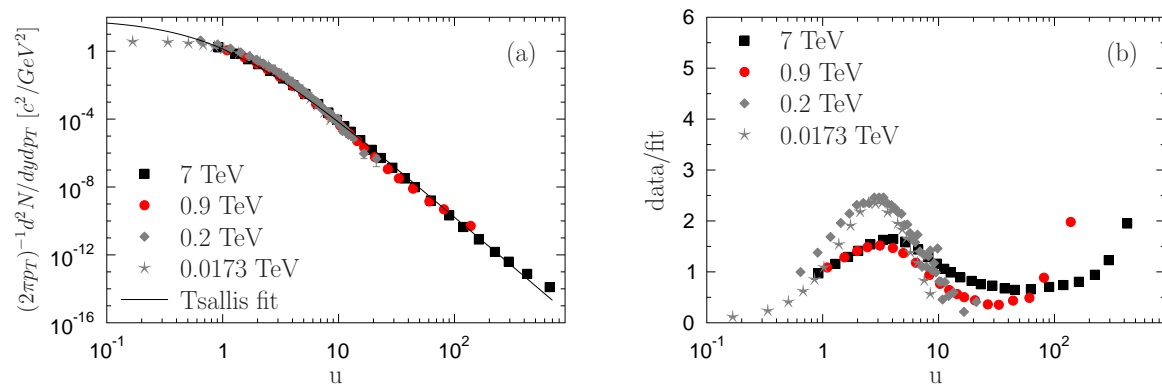


Figure 5. (Color online) (a) Data for transverse momentum distributions for different energies [17, 18, 13] plotted by using the scaling variable u defined by Eq. (12) and fitted by Eq. (3) in the variable u . (b) The ratio $data/fit$ for results presented in (a).

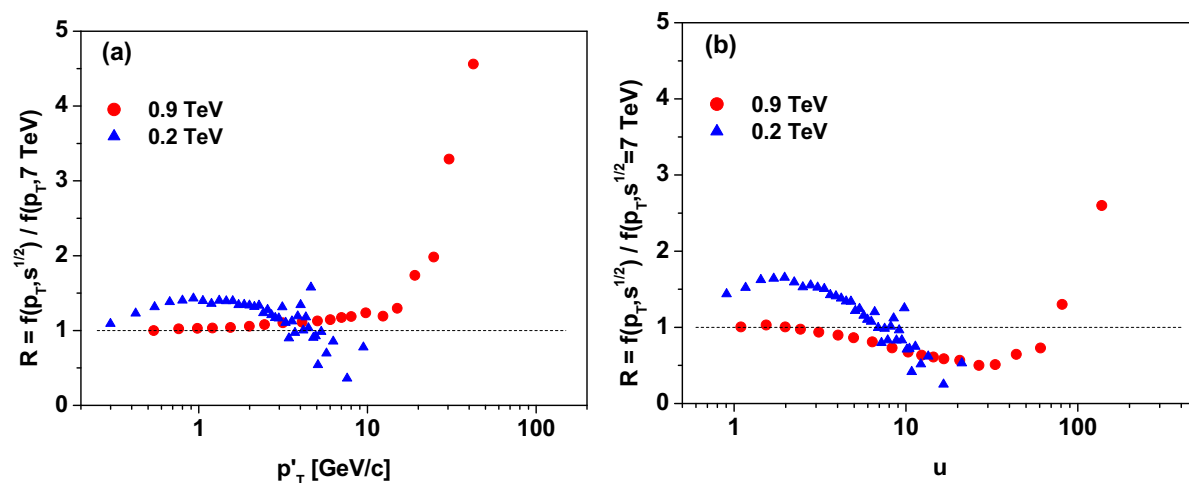


Figure 6. (Color online) Ratios of transverse momenta distributions at energies $\sqrt{s} = 0.9$ and 0.2 TeV with reference to the distribution at $\sqrt{s} = 7$ TeV, expressed in variable p'_T at (a) and variable u at (b).

similar, in a sense, to that proposed on a different occasion in [25]. It is therefore not necessarily connected with thermodynamics. In fact such T_{eff} also occurs in a description of the growth of the so called complex free networks if one associates p_T with the number of links [26]. When looking at hadron production from the perspective of stochastic networks [27] one can argue that the power law seen in transverse momenta spectra means that hadronization can be viewed as a process of formation of some specific network taking place in the environment of gluons and quark-antiquark pairs formed during the hadronization process. In this case their actual original energy-momentum distributions would be of secondary importance in comparison to the fact that, because of their mutual interactions, they connect to each other and that this

process of connection has its distinctive dynamical consequences¶.

More formally, following [26] observe that, whereas

$$\frac{df(x)}{dx} = -\frac{1}{T}f(x) \implies f(x) = \frac{1}{T} \exp\left(-\frac{x}{T}\right), \quad (15)$$

the x -dependent T in the form,

$$T \rightarrow T(x) = T_0 + (q-1)x, \quad (16)$$

results in a Tsallis distribution:

$$\frac{df(x)}{dx} = -\frac{1}{T_0 + (q-1)x}f(x) \implies f(x) = \frac{2-q}{T_0} \left[1 - (1-q)\frac{x}{T_0}\right]^{\frac{1}{1-q}}. \quad (17)$$

When fitting data with this distribution, one encounters the necessity to use in Eq. (17) the s -dependent q and T_0 :

$$q = q(s) = q_0 + q'(s) \quad \text{and} \quad T_0 = T_0(s) = T'_0 + T'(s). \quad (18)$$

To compensate for this s -dependence, one can modify the temperature in Eq. (17), for example by allowing for an x -dependence:

$$T_0 \rightarrow T_{eff}(x) = T'_0 - bx, \quad (19)$$

where the parameter b can be s -dependent. Returning to Eq. (16), one now has

$$T(x) = T'_0 + (q-1)x - bx \quad (20)$$

and that, solving the present form of Eq. (17), one finds

$$f(x) = \frac{2-q+b}{T_0} \left[1 - (1-q+b)\frac{x}{T_0}\right]^{\frac{1}{1-q+b}}. \quad (21)$$

Identifying now: $xb(s) = T'(s)$ and $b(s) = q'(s)$, one obtains an energy independent distribution

$$f(x) = \frac{2-q_0}{T'_0} \left[1 - (1-q_0)\frac{x}{T'_0}\right]^{\frac{1}{1-q_0}}. \quad (22)$$

In reality the situation is more complicated because here the x -dependence was introduced to T on the level of the distribution function, not in the differential equation.

¶ The possible line of reasoning is as follows: Suppose we start with some initial state consisting of a number n_0 of already existing $(q\bar{q})$ pairs (identified with vertices in the network). We add to them, in each consecutive time step, another vertex (a new $(q\bar{q})$ pair), which can have k_0 possible connections (links in the network language) to the old state. Assume that quarks are dressed by interaction with surrounding gluons and therefore "excited" and that each quark interacts with k other quarks (has k links). Assuming further that the "excitation" of a quark is proportional to the number of links k (which is proportional to the number of gluons participating in "excitation", i.e., existing in the vicinity of a given quark), the chances to interact with a given quark grow with the number of links k attached to it. The new links will be preferentially attached to quarks already having large k . This corresponds to building up a so called preferential network, which evolves due to the occurrence of new $(q\bar{q})$ pairs from decaying gluons.

Nevertheless, it seems that with such manipulations one can expect a distribution of the form

$$f(x) \propto \left[1 - (1 - q_{eff}) \frac{x}{T_{eff}} \right]^{\frac{1}{1-q_{eff}}} \quad (23)$$

in which the s -dependence should be noticeably reduced. This leads us to the variable u introduced in Eq. (12).

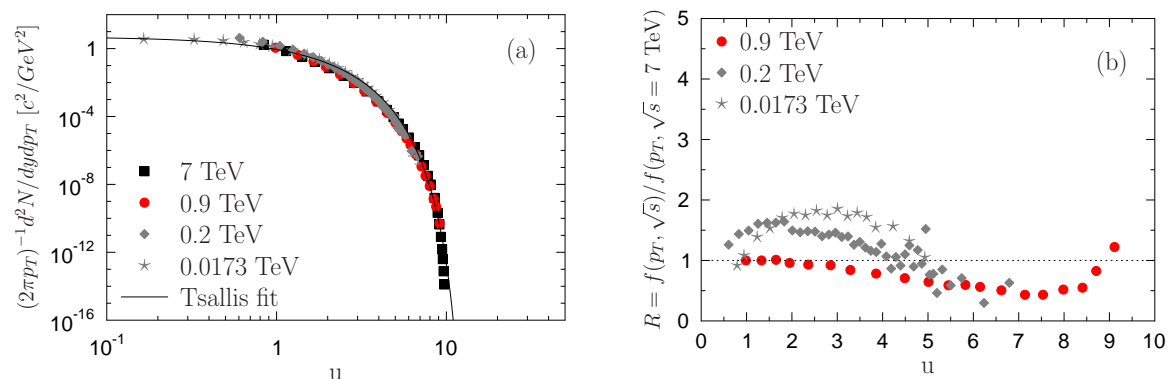


Figure 7. (Color online) (a) Data for transverse momentum distributions for different energies [17, 18, 13] plotted by using the scaling variable u defined by Eq. (12) with $b < 0$. (b) Ratios of transverse momenta distributions at energies $\sqrt{s} = 0.9, 0.2$ and 0.0173 TeV with reference to the distribution at $\sqrt{s} = 7$ TeV.

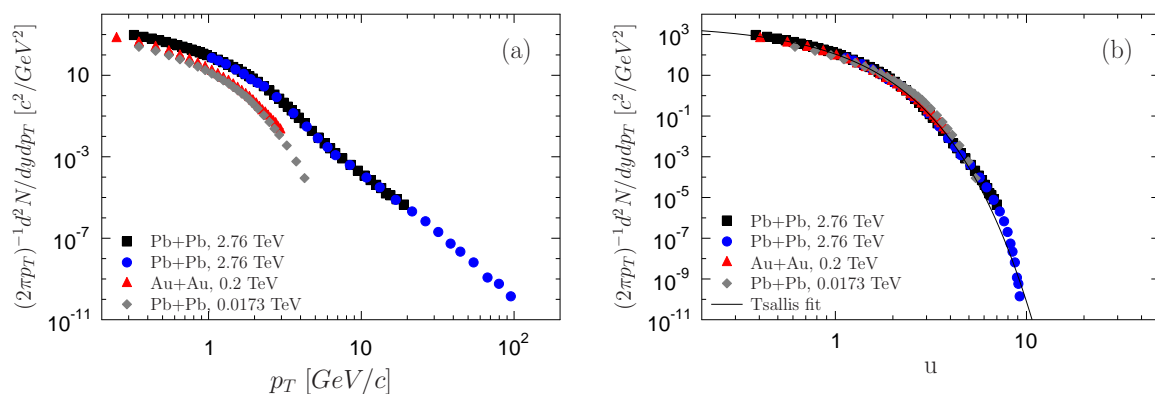


Figure 8. (Color online) (a) Data for transverse momentum distributions for central A+A collisions at different energies. Data for Pb+Pb at 2.76 TeV comes from ALICE [15] and CMS [28], data for Au+Au at 0.2 TeV come from PHENIX [29] and for Pb+Pb at 0.0173 TeV from NA49 [30]. (b) The same plotted by using the scaling variable u defined by Eq. (12) with $b < 0$.

Notice that, whereas for the choice of $b(s)$ used above one has $b > 0$ and T_{eff} was decreasing with p_T (increasing, in the network approach discussed above, action of the

preferential attachment) one can also choose a parametrization for which $b < 0$ and T_{eff} increases with p_T cancelling effects of "preferential attachment". For example, for $b(s) = -0.109 + 0.115(\sqrt{s})^{-0.3}$, one obtains a scaling of distributions in the variable u with the quasi-exponential form of the function $h(u)$ (i.e., in a Tsallis distribution with q close to 1). The corresponding results are shown in Fig. 7, together with the corresponding Tsallis fit for $q = 0.955$ (and $C = 5$, $u_0 = 0.6$).

Actually, the same kind of scaling is also possible for $A + A$ collisions for $b(s) = -0.052 - 0.0002(\sqrt{s})^{0.7}$. In Fig. 8 one can see distributions of p_T for central collisions (0 – 5% centrality) for different data together with the Tsallis fit for $q = 0.9999$ (and $C = 2800$, $u_0 = 0.32$).

To summarize: it is possible to fit all available data on p_T distributions using some universal, energy independent, parameter q . Therefore there is a possibility of q -scaling. This can be done by choosing a variable u defined in Eq. (12) in the distribution $h(u)$ given by Eq. (3). Scaling can be achieved either by increasing q to $q = 1.172$ (for $b > 0$, cf., Fig. 5) or by decreasing it to $q \sim 1$ (for $b < 0$, cf., Fig. 7; in this case Eq. (3) almost coincides with Eq. (1))⁺. The observed Tsallis distributions do not necessarily indicate thermalization of the system considered (there are numerous examples of non-thermal sources of Tsallis distributions, cf. [7]). The possible explanation we propose is based on the description of a hadronization process in analogy with complex free networks [27]. Alternatively, one can interpret Eq.(17) as a stationary solution of the Fokker-Planck equation corresponding to a Langevin equation with multiplicative noise with nonzero mean value $\langle \eta(t) \rangle = 1 - q$ [31] (cf. also [7]). The possible connection with QCD based ideas [25] is also indicated (but this would demand special attention, which is outside of the scope of this paper).

Acknowledgment

Acknowledgment: Partial support (GW) of the Ministry of Science and Higher Education under contract DPN/N97/CERN/2009 is gratefully acknowledged. We would like to warmly thank Dr Eryk Infeld for reading this manuscript.

References

- [1] Hagedorn R 1965 *Nuovo Cim. Suppl.* **3**, 147
- [2] Hagedorn R 1984 *Riv. Nuovo Cime.* **6** (No 10), 1
- [3] Michael C and Vanryckeghem L 1977 *J. Phys. G* **3** L151 (1977); Michael C 1979 *Prog. Part. Nucl. Phys.* **2**, 1
- [4] Arnison G et al. (UA1 Collab.) 1982 *Phys. Lett. B* **118**, 167

⁺ Notice that only parameter q in Eq. (3), i.e., for variable p_T , has physical sense. In this case it can, for example, be connected with temperature fluctuations, cf., Eq. (5). After rescaling the variable (i.e., when using u instead of p_T) q changes its meaning and can be both greater or smaller than all the experimentally observed values.

- [5] Tsallis C 1988 *Stat. Phys.* **52**; 2009 *Eur. Phys. J. A* **40**, 257 and *Introduction to Nonextensive Statistical Mechanics* (Springer, 2009). For an updated bibliography on this subject, see <http://tsallis.cat.cbpf.br/biblio.htm>.
- [6] Wilk G and Włodarczyk Z 2009 *Eur. Phys. J. A* **40**, 299; 2012 *Cent. Eur. J. Phys.* **10**, 568
- [7] Wilk G and Włodarczyk Z, *Consequences of temperature fluctuations in observables measured in high energy collisions*, arXiv:1203.4452, to be published in 2012 *Eur. Phys. J. A*
- [8] Wilk G and Włodarczyk Z 2000 *Phys. Rev. Lett.* **84**, 2770
- [9] Ming Shao, Li Yi, Zebo Tang, Hongfang Chen, Cheng Li and Zhangbu Xu 2010 *J. Phys. G* **37**, 085104
- [10] Cleymans J and Worku D 2012 *J. Phys. G* **39**, 025006
- [11] Adare A et al. (PHENIX Collaboration) 2011 *Phys. Rev. D* **83**, 052004
- [12] Abelev B I et al. (STAR Collaboration) 2007 *Phys. Rev. C* **75**, 064901 (2007)
- [13] Khachatryan V et al. (CMS Collaboration) 2010 *JHEP***02**, 041 and 2010 *Phys. Rev. Lett.* **105**, 022002
- [14] Aad G et al., (ATLAS Collaboration) 2011 *New J. Phys.* **13**, 053033 053033.
- [15] Aamodt K et al. (ALICE Collaboration) 2010 *Phys. Lett. B* **693** and 2011 *Eur. Phys. J. C* **71**, 1594 and 1655
- [16] Iannicu E and Triantafyllopoulos D N 2005 *Nucl. Phys. A* **756**, 419; Soyez G 2005 *Phys. Rev. D* **72**, 016007; Wei Zhu, Zhenqi Shen and Jianhong Ruan 2008 *Chi. Phys. Lett* **25**, 3605 [arXiv:0809.0609]
- [17] Alt C et al. (NA49 Collaboration) 2006 *Eur. Phys. J. C* **45**, 343; Anticic T et al. (NA49 Collaboration) 2004 *Phys. Rev. C* **70**, 034902
- [18] Albajar C et al. (UA1 Collab.) 1990 *Nucl. Phys. B* **335**, 261
- [19] Wibig T 2010 *J. Phys. G* **37**, 115009
- [20] Barnaföldi G G, Ürmösy K and Biró T S 2011 *J. Phys. Conf. Ser.* **270** 012008
- [21] McLerran L and Praszalowicz M 2010 *Acta Phys. Polon B* **41**, 1917 and 2011 *Acta Phys. Polon B* **42**, 99
- [22] Praszalowicz M 2011 *Acta Phys. Polon. B* **42**, 1557
- [23] Praszalowicz M 2011 *Phys. Rev. Lett.* **106**, 142002
- [24] Koba Z, Nielsen H B and Olesen P 1972 *Nucl. Phys. B* **40** 319
- [25] Wong C-Y, 2011 *Phys. Rev. C* **84**, 024901
- [26] Wilk G and Włodarczyk Z 2004 *Acta Phys. Polon. B* **35**, 871 and 2005 *Acta Phys. Polon. B* **36**, 2513; Tsallis C 2008 *Eur. Phys. J. Special Topics* **161**, 175
- [27] Wilk G and Włodarczyk Z 2004 *Acta Phys. Polon. B* **35**, 2141
- [28] Kurt P et al. (CMS Collaboration) 2012 *J. Phys. Conf. Series* **347**, 012004
- [29] Adler S S et al. (PHENIX Collaboration) 2010 *Phys. Rev. C* **81** 034911
- [30] Alt C et al. (NA49 Collaboration) 2008 *Phys. Rev. C* **77**, 024903
- [31] Biró T S and Jakovác A 2005 , *Phys. Rev. Lett.* **94**, 132302